SEQUENTIAL APPROXIMATE OPTIMIZATION OF COMPOSITE STRUCTURES USING RADIAL BASIS FUNCTIONS

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https://doi.org/10.21452/bccm4.2018.16.10

Abstract

The use of optimization techniques is necessary in order to explore the full potential of laminated composite structures. Unfortunately, the computational cost of the optimization process can be very high when numerical methods are used to carry out the structural analysis. This work addresses the use of surrogate models to reduce the computational cost to optimize composite structures. The PSO algorithm is used for optimization and a sequence of surrogate models, based on the use of Radial Basis Functions, is used to approximate the structural responses. The accuracy of the proposed approach is assessed using a set of laminate optimization problems and very good results were obtained.

1. INTRODUCTION

Fiber reinforced composite (FRC) materials present high resistance/weight and stiffness/weight ratios, corrosion and fatigue resistance, and other interesting properties for high performance structural applications. These composites are formed from high strength fibers embedded in a polymeric matrix, resulting in an orthotropic composite material. Typically, several layers with different fiber orientations are stacked to obtain more efficient designs, leading to a laminated structure. Due to their complex mechanical behavior, the analysis of laminate structures requires the use of numerical methods, as the Finite Element Method (FEM) and Isogeometric Analysis (IGA).

The design of laminated structures requires the determination of the number of layers and the characteristics of each layer (material, thickness and fiber orientation). Since there are a large number of possibilities, the use of optimization techniques is necessary in order to explore the full potential of laminated composite structures. However, the optimization of laminated composite structures has a high computational cost, especially when heuristic methods, as Genetic Algorithm or Particle Swarm Optimization (PSO), are applied. An alternative to reduce the processing time is to use surrogate models to approximate the structural responses.

Surrogate models build an approximation of the structure behavior based on the structure responses evaluated by the numerical methods at a set of selected sampling points. Artificial

Neural Networks (ANN), Radial Basis Functions (RBF), Support Vector Regression (SVR) and Kriging are some surrogate models widely used (Wang and Shan, 2007; Forrester *et al.*, 2008). The RBF approach is used in this work, since it stands out for its simplicity, accuracy and robustness, when compared with other options (Jin et al., 2001).

The PSO algorithm is used for optimization since it is simple, efficient and can easily handle discrete variables (Barroso et al., 2017). A Sequential Approximate Optimization (SAO) approach is presented, where a sequence of RBF models is used to approximate the laminate responses. The accuracy of the proposed approach is assessed using a set of laminate optimization problems and very good results were obtained.

2. RADIAL BASIS FUNCTIONS

RBF models where proposed by Hardy (1971) to interpolate geographic data. Nowadays, this technique is widely used as a surrogate model (Jin et al., 2001; Forrester *et al.*, 2008; Kitayama *et al.*, 2011; Amouzgar and Strömverg, 2016). An RBF model can be written as:

$$\hat{f}(\mathbf{x}) = \sum_{i=1}^{n} w_i \varphi(||\mathbf{x} - \mathbf{c}_i||) \tag{1}$$

w_i are known as weights, φ are the Radial Basis Functions, $\mathbf{c}_i \in \mathbb{R}^m, i = 1, 2, \ldots, n$ are the basis centers, which can be taken from the sampling points by different techniques (Haykin, 2008; Amouzgar and Strömberg, 2016). In this work, all the sampling points are RBF centers.

The RBF herein used was the Gaussian function

$$\varphi(r) = \exp\left(-\frac{r^2}{\sigma^2}\right) \tag{2}$$

where $r = ||\boldsymbol{x} - \boldsymbol{c}||$ is the radial distance and σ is a parameter which controls the RBF shape. This parameter can be evaluated by cross-validation (Forrester *et al.*, 2008) or by closed-form expressions depending on the distance between the sampling points (Kitayama *et al.*, 2011).

The simplest way to create an RBF model is by interpolating the sampling points $f(\mathbf{x}_i) = \mathbf{y}_i$:

$$\begin{bmatrix} \varphi_{11} & \varphi_{12} & \ldots & \varphi_{1n} \\ \varphi_{21} & \varphi_{22} & \cdots & \varphi_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \varphi_{n1} & \varphi_{n2} & \cdots & \varphi_{nn} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \Rightarrow \mathbf{H}\,\mathbf{w} = \mathbf{y} \tag{3}$$

where $H_{ij} = \varphi\left(\left||\mathbf{x}_i - \mathbf{x}_j|\right|\right), i, j = 1, 2, \ldots, n$. The interpolation matrix is nonsingular provided that the sampling points $(\mathbf{x}_i)$ are distinct (Forrester *et al.*, 2008). However, this matrix tends to become bad conditioned when the sampling points are close to each other (Haykin, 2008). In addition, overfitting, in which the model only represents well the points included in the sampling points, can occur when many points are used (Forrester *et al.*, 2008). Both problems can be avoided by finding the weights w_i using the least squares approach considering a regularization parameter (λ) (Kitayama *et al.*, 2011):

$$E = \sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}} \left(\mathbf{y}_i - \hat{f}(\mathbf{x}_i)\right)^{\mathbf{2}} + \sum_{\boldsymbol{j}=\mathbf{1}}^{\boldsymbol{n}} \lambda\, w_{\boldsymbol{j}}^{\mathbf{2}} \tag{4}$$

The error (E) minimization yields the linear system:

$$(\mathbf{H}^{\mathrm{T}}\mathbf{H} + \lambda\mathbf{I})\mathbf{w} = \mathbf{H}^{\mathrm{T}}\mathbf{y} \tag{5}$$

whose solution yields the weight vector ($\mathbf{w}$). The value $\lambda = 10^{-3}$ was adopted in this work.

3. SEQUENTIAL APPROXIMATE OPTIMIZATION

The simplest approach to surrogate based optimization is to build a fixed surrogate model based on an initial sampling plan. Different Design of Experiments (DoE) techniques, as the Latin Hipercube Sampling and Hammersley Sequence Sampling (HSS), can be used to generate this sampling plan (Forrester *et al*., 2008; Amouzgar and Strömberg, 2016). The fixed surrogate is used to in all optimization iterations (or generations in GA). This approach has been successfully applied to several problems, but it requires a large sampling plan, which can be costly for problems with many design variables. Since the optimum solution is not known, costly numerical analysis are carried out to build an accurate surrogate model in regions far away from the optimum.

A better alternative is to use the Sequential Approximate Optimization (SAO) approach (Schmit and Farshi, 1974; Haftka and Gürdal, 1991; Jones *et al.*, 1998; Kitayama *et al*., 2011; Pan *et al*., 2014; Chung *et al.*, 2018). In this approach, an initial surrogate model is built based on a small sampling plan. This initial model is updated after each iteration of the optimization algorithm, including the optimum solution found in this iteration and possibly other new sampling points. This approach naturally generates more sampling points, and a better approximation, in the region close to the optimum. However, it can be shown that including only the optimum solution found at each iteration is not sufficient to find the global optimum solution and different infill strategies have been proposed to update the surrogate model (Forrester *et al*., 2008).

In this work, the initial sampling is generated by the Hammersley Sequence Sampling (Amouzgar and Strömberg, 2016), the design optimization is carried-out is a Hybrid PSO-GA algorithm including special laminate operators (Barroso et al., 2017) and the surrogate model is updated using the infill strategy proposed by Kitayama *et al*. (2011).

In order to balance the local and global approximation aspects, after each iteration not only the current optimal solution, but also new sampling points are evaluated using numerical methods and used to update the surrogate model. These points are found minimizing the density function (Kitayama *et al*., 2011) in order to include new sampling points where the model is poorly approximated. It is expected that adding new sampling points in sparse regions of the design space will lead to convergence to the global optimum (Kitayama *et al.*, 2011).

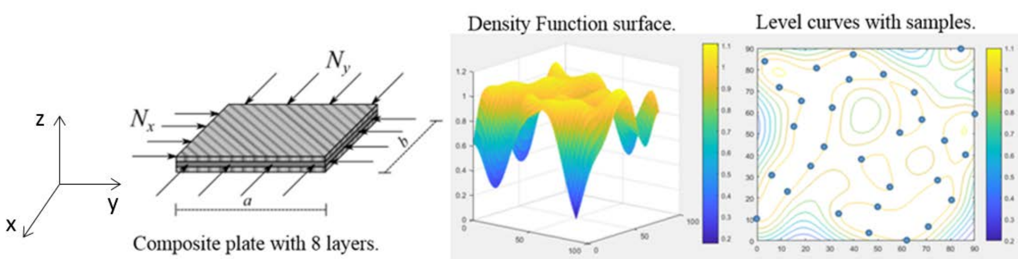

Figure 1. Density Function surface and level curves.

The density function is approximated by a RBF model whose weights ($\mathbf{w}^{\mathrm{DF}}$) are evaluated using Equation (*5*) with $\mathbf{y}^{\mathrm{DF}} = [1, 1, 1, \ldots, 1]^{T}_{mx1}$. This function is illustrated in Figure 1 for a sampling plan composed 30 points (blue circles) used to approximate the buckling load of a simply support square plate with balanced and symmetric layup subjected to a biaxial load. The plate has 8 plies corresponding to two design variables (θ_1, θ_2). It can be noticed that minimizing this function results in the insertion of the point **x** = [90, 0] in the sample.

In this work, the minimum of the density function is found using the standard PSO algorithm. The number of infill points to be included at each iteration is taken as $n/2$ (Kitayama *et al*., 2011). The general SAO algorithm used in this work is presented in Figure *2*. For practical problems, the costlier step is the evaluation of the sampling points, since it involves the analysis of the composite structure by numerical methods.

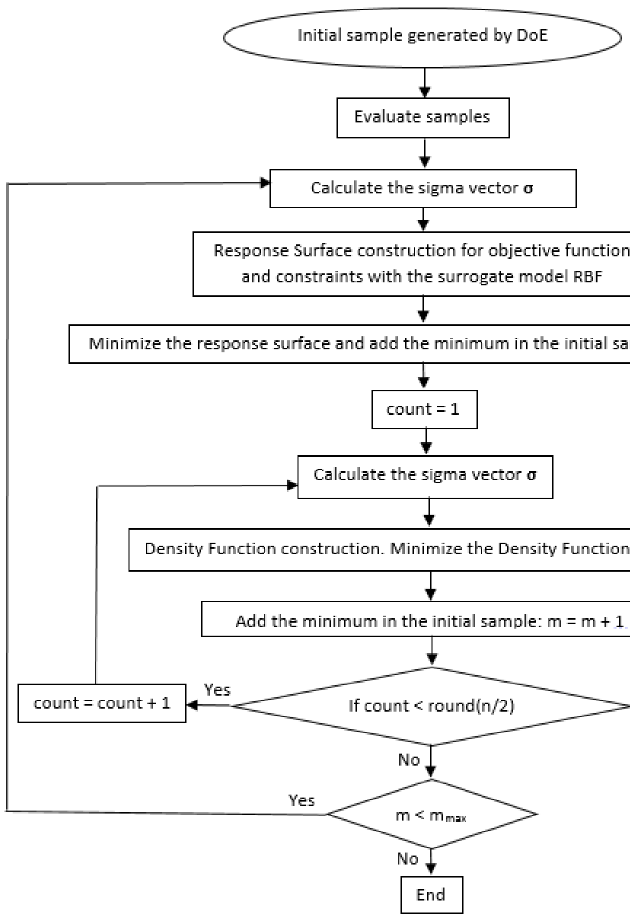

Figure 2. SAO flowchart.

4. RESULTS

The first example consists on a square laminated plate, with 8 plies, simply supported and subjected to biaxial loading (Figure 1). The design variables are the fiber orientation of each ply. Since the layup is symmetric and balanced, there are only two design variables ($0 \leq \theta_1, \theta_2 \leq 90$). The objective is to find the fiber orientations ($\mathbf{x} = [\theta_1, \theta_2]$) that maximize the buckling load (λ_b):

$$\begin{array}{ll} \text{Find} & \mathbf{x} = [\theta_1, \theta_2] \text{ that} \\ \text{maximize} & \lambda_b \\ \text{with} & 0° \leq \theta_i \leq 90° \end{array} \qquad (6)$$

Table 1 presents the geometrical parameters and material properties. Reddy (2004) presents a closed-form expression for the buckling load, which is exact for cross-ply laminates and give accurate results for angle-ply laminates with several plies. This solution will be adopted as the exact structural response to build the surrogate model.

Table 1. Geometry and properties of Example 1.

Geometry (m)			Material: Carbon-Epoxy			
a	b	thickness	$E_1(GPa)$	$E_2(GPa)$	$G_{12}(GPa)$	ν_{12}
0.508	0.508	1.272e-4	130.71	6.36	4.18	0.32

An initial sample with 9 points was generated using the Hammersley Sequence Sampling. The optimization was carried out using 5 iterations and a swarm composed of 100 particles. The final sample has 19 points due to the addition of 2 points at each iteration. The results presented in Table *2* show that the same optimum layup was obtained by the exact and SAO approaches, with the former requiring 500 structural analysis and the latter only 19.

Table 2. SAO optimization results.

Approach	Layup	λ_b
Exact	$[\pm 45\ \pm 45]_s$	462.63
SAO	$[\pm 45\ \pm 45]_s$	462.63

The second problem corresponds to a simply support rectangular plate under biaxial loading (Figure 1) with N_x/N_y = 0.125 (Barroso et al., 2017). Table 3 presents the geometry and properties of this plate. Since a symmetric and balanced layup with 48 plies is adopted, the problem has 12 design variables. The objective is to find the best layup ($\mathbf{x} = [\theta_1, \theta_2, \ldots, \theta_{12}]$) that maximize the plate strength, considering buckling (λ_b) and material failure (λ_f) according to the Maximum Strain Criterion (Daniel and Ishai, 2004). The number of contiguous plies (N_{cp}) is limited to 4.

$$\begin{array}{ll} \text{Find} & \mathbf{x} = [\theta_1, \theta_2, \ldots, \theta_{12}] \text{ that} \\ \text{maximize} & \lambda_b, \lambda_f \\ \text{subjected to} & 1 - \frac{N_{cp}}{cp_{max}} \leq 0 \\ \text{with} & 0° \leq \theta_i \leq 90° \end{array} \quad (7)$$

Table 3. Plate properties and geometry.

Geometry (m)			Engineering properties				Ultimate strain		
a	b	*thickness*	$E_1(GPa)$	$E_2(GPa)$	$G_{12}(GPa)$	ν_{12}	ε_1^u	ε_2^u	γ_{12}^u
0.508	0.127	1.27e-4	138	9.0	7.1	0.3	0.008	0.029	0.015

The initial sample used in SAO has 137 points. This problem was optimized initially using 100 particles and 20 iterations. Since the problem has 12 variables, 7 points were added to the sample at the end of each iteration, resulting in a final sample with 277 points. The obtained results are presented in Table *4*. It can be noted that exact and SAO layups are not the same, but the difference in the buckling load (λ_b) was only -2.11%, on the other hand the difference in failure criterion was 12.18%.

Table 4. Optimization results for 20 and 50 iterations.

Approach	Layup	λ_b	λ_f
Kogiso et al. (1994)	$[\pm45_5\ 0_4\ \pm45\ 0_4\ 90_2\ 0_2]_s$	14659.58	13518.66
SAO (20 iterations)	$[\pm45_5\ 0_2\ \pm45\ 0_2\ 90_2\ 0_4\ \pm45]_s$	14969.50	11871.10
SAO (50 iterations)	$[\pm45_5\ (0_4\ \pm45)_2\ 0_2]_s$	14680.00	13458.30

Better results were obtained optimizing the problem using 50 iterations, as shown in Table *4*. The exact and SAO layups are still different, but the buckling load difference decreases to only 0.13% and the failure load difference decreases to only 0.44%. Therefore, SAO results can be improved increasing the number of iterations, since more sampling points are included, leading to a better surrogate model. Obviously, the computational cost also increases due to the additional structural analysis.

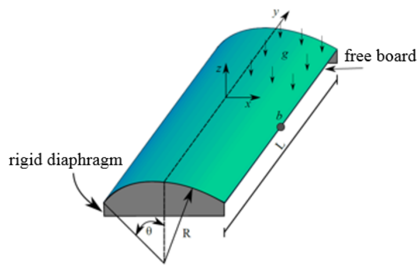

Figure 3. Laminated cylindrical shell.

The third problem consists in the stiffness maximization of a laminated cylindrical shell (Figure 3) with 40 plies (Barroso, 2015). Table 5 presents the shell properties. The problem has 10 design variables since a symmetric and balanced layup was adopted. The shell stiffness can be maximized by minimizing the vertical displacement (w) of point b. In addition, the required safety factor (SF) is equal to 1.5. The number of contiguous plies (N_{cp}) is limited to 4. The optimization problem is written as:

$$\begin{aligned} &\text{Find} && \mathbf{x} = [\theta_1, \theta_2, \dots, \theta_{10}] \quad \text{that} \\ &\text{minimize} && w_b \\ &\text{subjected to} && \frac{N_{cp}}{cp_{max}} - 1 \le 0 \\ & && 1 - \frac{S_{TW}}{SF} \le 0 \\ &\text{with} && 0° \le \theta_i \le 90° \end{aligned} \qquad (8)$$

The Isogeometric Analysis (IGA) was used to evaluate the displacements and stresses considering a 3D model. The Tsai-Wu criterion was adopted to evaluate the safety factor against material failure (S_{TW}).

Table 5. Geometrical and engineering properties.

g (kN/m²)	R (m)	L (m)	θ (°)	t (cm)	N_{lam}	t_{lam} (cm)	E_1 (GPa)
45	3.0	6.0	40	3.0	40	0.075	147
E_2 (GPa)	E_3 (GPa)	ν_{12}	ν_{13}	ν_{23}	G_{12} (GPa)	G_{13} (GPa)	G_{23} (GPa)
10.3	10.3	0.27	0.27	0.54	7.0	7.0	3.7

The reference solution, where all designs were analyzed using IGA, was obtained using 49 particles and 30 iterations. The SAO method used the same number of particles, but considered initially only 20 iterations. The initial sample had 99 points. Since the problem has 10 variables, 6 points were added to the sample at the end of each iteration, resulting in a final sample with 219 points. The results are presented in Table 6.

Different layups were obtained by the exact and SAO approaches, but the differences were negligible for the displacement and safety factor. Increasing the number of SAO iterations to 30 lead to even closer results. In this case, the use of SAO leads to a reduction of 82.77% in the elapsed time. Therefore, SAO makes feasible the use of optimization techniques in the design of complex laminated structures.

Table 6. SAO optimization results.

Approach	Layup	Time	w_b	S_{TW}
IGA (30 iterations)	$[(90_4\ \pm 45)_2\ 0_4\ \pm 45\ 0_2]_s$	12306 s	11.58	2.12
SAO (20 iterations)	$[\pm 45\ 90_4\ \pm 45_3\ 0_4\ \pm 45\ 0_2]_s$	1342 s	12.09	2.05
SAO (30 iterations)	$[90_2\ (90_2\ \pm 45)_2\ \pm 45\ 0_4\ \pm 45\ 0_2]_s$	2120 s	11.63	2.10

5. CONCLUSION

This work studied the use of Sequential Approximate Optimization (SAO) to laminate composite structures. In this SAO approach, the HSS sampling technique was used to generate the initial sample and Radial Basis Functions (RBF) were used as a surrogate model. The surrogate model is updated at each iteration including the iterative optimum solution and additional points in regions sparsely sampled of the design space. The additional sampling points are found minimizing a RBF density function using the PSO algorithm.

Optimization problems with both small and large number of design variables were solved and very good results were found. The efficiency of the SAO was demonstrated by the large reduction in the optimization time obtained when numerical methods are required for structural analysis. The accuracy of SAO results can be increased using a large number of optimization iterations. Therefore, the obtained results depend on the available computer resources and project schedule constraints. Thus, SAO is a promising technique to allow the design optimization of real-world laminated composite structures.

ACKNOWLEDGEMENTS

The support given by Elias Barroso in the implementation of the Isogeometric Analysis problem, the help provided by Guilherme Mendes in the use of the program BIOS, the Laboratório de Mecânica Computacional e Visualização infrastructure to perform this work, and the financial support by CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior) are gratefully acknowledged.

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